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THE INFLUENCE OF BOUNDARY CONDITIONS ON THE BUCKLING OF STIFFENED CYLINDRICAL SHELLS

Josef Singer, et al

Technion - Israel Institute of Technology

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Theoretical and experimental studies on the influence of boundary conditions on the buckling of stiffened cylindrical shells and their vibrations are discussed. The effect of prebuckling deformations on the buckling loads and vibrations of stiffened shells is studied and compared with that in the case of unstiffened shells. The in-plane boundary conditions are found to be of particular importance for stiffened cylindrical shells and their effect differs significantly from that in unstiffened shells. The effect of axial restraints, which are found to be of prime importance in stringer-stiffened shells are also studied. By

correlation with the vibration tests on the same shells a method is developed for definition of the actual boundary conditions of a stiffened shell non-destructively. The effect of eccentricity of loading on stringer-stiffened shells is studied experimentally and correlated with vibration tests on the same shells. Priliminary results of a non-destructive experimental method for prediction of buckling loads based on vibration testing of stiffened shells are also presented.

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THE INFLUENCE OF BOUNDARY CONDITIONS ON THE BUCKLING OF STIFFENED CYLINDRICAL SHELLS

by
Josef Singer and Aviv Rosen

Department of Aeronautical Engineering
Technion - Israel Institute of Technology
Haifa, Israel

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ABSTRACT

Theoretical and experimental studies on the influence of boundary condition on the buckling of stiffened cylindrical shells and their vibrations are discussed. The effect of prebuckling deformation on the buckling loads and vibrations of stiffened shells is studied and compared with that in the case of unstiffened shells. The in-plane boundary conditions are found to be of particular importance for stiffened cylindrical shells and their effect differs significantly from that in unstiffened shells. Axial restraints are found to be of prime importance in stringer-stiffened shells, and therefore the effect of elastic axial restraints is also studied.

By correlation with the vibration tests on the same shells a method is developed for definition of the actual boundary conditions of a stiffened shell non-destructively. The effect of eccentricity of loading on stringer-stiffened shells is studied experimentally and correlated with vibration tests on the same shells. Preliminary results of a non-destructive experimental method for prediction of buckling loads based on vibration testing of stiffened shells are also presented.

TABLE OF CONTENTS

LIST OF SYMBOLS

A_1, A_2	-	cross section of stringer and ring, respectively.
a	-	ring spacing (distance between centers of rings).
b_1	-	stringer spacing (distance between centers of stringers).
C1,C2,C3,C4	-	notation for clamped boundary conditions.
c_1	-	width of stringer.
d_1	-	height of stringer.
E	-	Young's modulus.
e_1, e_2	-	stringer or ring eccentricity, respectively (distance from shell middle surface to stiffener centroid).
$\bar{e}$	-	eccentricity of loading (distance from shell middle surface to the point of application of load).
f	-	frequency
h	-	thickness of shell
I_{11}, I_{22}	-	moment of inertia of stringer or ring stiffener cross-section about its centroidal axis.
k_1	-	axial elastic restraint
k_4	-	rotational elastic restraint
L	-	length of shell.
M_x	-	moment resultant in axial direction.
m	-	number of half longitudinal waves.
N_x	-	axial membrane force resultant.
N_{xy}	-	shear membrane force resultant.
n	-	circumferential wave number.
P	-	compressive axial load.
P_{CR}	-	theoretical buckling load.

P_{PRE} - theoretical buckling load with nonlinear prebuckling deformations considered.

$P_{membrane}$ - theoretical buckling load with prebuckling deformation neglected.

P_{EXP} - experimental buckling load.

R - radius to shell middle surface.

SS1,SS2,SS3,SS4 - notation for simply supported boundary conditions.

u,v,w - displacements in axial, circumferential and radial directions respectively (radial direction positive inward).

x - axial coordinate.

Z - $(1-\nu^2)^{1/2}(L/R)^2(R/h)$, Batdorf shell parameter.

ν - Poisson's ratio.

ρ_{PRE} - $P_{PRE}/P_{membrane}$.

ρ - $P_{EXP}/P_{membrane}$, "linearity".

η_{ti} - torsional stiffness parameter of stringer (see [32] or [53]).

ϕ - circumferential coordinate.

Subscripts following a comma indicate differentiation.

LIST OF FIGURES

1. INTRODUCTION

The importance of the boundary conditions in determining the buckling behavior of shells has been given serious attention only in the last decade. Though some of the earlier studies took boundary effects into account, they usually did it only partially and the correlation with test conditions was generally rather vague. In the beginning of the sixties, much effort was devoted to theoretical studies on the influence of in-plane boundary conditions as well as prebuckling deformations on the buckling of cylindrical shells under axial compression (see for example [1]-[9]), since axial compression was the most striking case of large discrepancies between predictions of classical buckling theory and experiment. Concurrently, extensive efforts were applied to more sophisticated and careful experiments with emphasis on the separation of the influence of various parameters (see for example [10]-[19]). Hence, the boundary conditions in these tests were usually reasonably well defined. Most investigators employed nearly fully clamped boundary conditions in their experimental studies, since these appeared to be most convenient for definition.

In the second half of the decade, the emphasis in both theoretical and experimental research shifted to the study of the effect of initial geometrical imperfections, since they were accepted to be the main degrading factor on the buckling loads of cylindrical shells. The many investigations following this trend, and developing the initial postbuckling analysis first proposed by Koiter in 1945 [20], have recently been reviewed in a survey of postbuckling theory by Hutchinson and Koiter [21]. The importance attributed to geometrical imperfections by all investigators in the field is evident in the survey. This emphasis, however, appears to have overshadowed the equally important problem of the influence of the boundary conditions. Indeed, except for some solitary voices (like Ref.[21] or [23]),the boundary conditions have been categorically relegated to second place by most researchers (see for example [19],[24] or [23]).

Only very recently has there been renewed interest in the effect of boundary conditions on buckling of cylindrical shells under axial compression (for example [26] or [27]) and other loads (for example [28]), as well as for vibrations (for example [29] or [30]), though for vibrations the influence of boundary conditions has always been kept in the forefront.

For stiffened shells, and in particular closely stiffened shells (for which local panel buckling is rarely critical) the effect of geometric imperfections is less pronounced. Hence the reduction in predicted buckling loads and scatter of test results is less severe, provided the boundary conditions are adequately accounted for. The author has for some years now defended the earlier statement by Van der Neut [31] that linear theory is adequate for prediction of buckling loads in shells with closely spaced stiffeners. Extensive experimental evidence for integrally ring-and-stringer-stiffened cylindrical shells (reviewed in [32]), confirms the applicability of linear theory at least as a first approximation, and with at least the same reliability as for isotropic cylindrical shells under external pressure. The influence of the boundary conditions is however, found to be of prime importance in stiffened shells and sometimes it even overshadows the effect of geometrical imperfections. The realization of this fact has motivated also the recent theoretical and experimental studies by the authors,aimed at better understanding of the influence of boundary conditions and their more precise assessment.

A brief review of the available information on the three boundary effects (prebuckling deformations, in-plane boundary conditions and out-of-plane boundary conditions) precedes the discussion of the present studies.

2. EFFECT OF PREBUCKLING DEFORMATIONS

The effect of prebuckling deformations, caused by the edge constraints, has been extensively studied for isotropic cylindrical shells under axial compression. The analyses have shown reductions in predicted buckling loads of 15-20 per cent for "simple supports" (zero rotational restraints - [4] [7][8] and [27]) and 8-10 per cent for clamped edges ([4],[23] and [27]). The magnitude of the effect of prebuckling deformations depends also considerably on the shell geometry parameters (L/R) and (R/h), or on their combination, the Batdorf parameter $Z = \sqrt{1-\nu^2}(L/R)^2\,(R/h)$, and on the in-plane boundary conditions. This dependence has recently been summarized in the very complete parametric study of Yamaki and Kodama [27]. Note that the notation used in the present paper for the in-plane boundary conditions:

SS1:	$w = M_x = N_x = N_{x\phi} = 0$	C1:	$w = w_{,x} = N_x = N_{x\phi} = 0$
SS2:	$w = M_x = u = N_{x\phi} = 0$	C2:	$w = w_{,x} = u = N_{x\phi} = 0$
SS3:	$w = M_x = N_x = v = 0$	C3:	$w = w_{,x} = N_x = v = 0$
SS4:	$w = M_x = u = v = 0$	C4:	$w = w_{,x} = u = v = 0$

follows [1] and [2] but differs from that used in [4] or [27]. There are 4 ranges of Z with regards to the effect of prebuckling deformations [27]: For very short cylinders when $Z \le 4$ the effect is negligible. For $4 < Z \le 30$ there is an increase in the buckling load compared to that obtained with prebuckling deformations neglected ($\rho_{PRE} = p_{PRE}/p_{membrane}$) , and then for $Z>30$. ρ_{PRE} is reduced in one case even to below 0.8. In the range $4 < Z < 400$ the effect varies noticeably with Z, whereas for $Z > 400$ it becomes almost constant irrespective of Z. Table 1, reproduced from [27], gives the magnitudes of the effect for different in-plane boundary conditions in this region of Z.

Table 1: Values of ρ_{PRE} for Isotropic Cylindrical Shells when $Z > 400$.

	Simple Supports			Clamped Edges			
B.C.	SS3	SS4		C1	C2	C3	C4
ρ_{PRE}	0.83	0.85		0.90	0.92	0.90	0.92

For the "weak-in-shear" SS1 and SS2 boundary conditions, which yield half the classical buckling load without consideration of prebuckling, the effect of prebuckling displacements is always negligible. In passing, one may also note that for isotropic cylinders under external pressure [33] the effect of prebuckling deformations is important only in short shells, whereas for $Z > 100$ it is negligible for all 8 in-plane boundary conditions.

The effect of nonlinear prebuckling deformations on the buckling of stiffened shells has received less attention. For ring-stiffened shells under axial compression the effect has been found to be similar to that in isotropic shells, but considerably smaller ([34] and [35]). For example, for a typical ring-stiffened shell tested in [35] AR5, with $(L/R)=1.31$, $(R/h)=477$ and hence $Z=782$, $(A_2/ah)=0.315$, $(e_2/h)=-1.54$ and $(I_{22}/ah^3)=0.127$, the ratio of the buckling load with prebuckling deformations taken into account to the value obtained by linear theory:

ρ_{PRE} = for SS3 B.C.'s

$\rho_{PRE} = 0.983$ for C4 B.C.'s

whereas for a corresponding isotropic shell [27] it would be:

$\rho_{PRE} = 0.844$ for SS3 B.C.'s

$\rho_{PRE} = 0.926$ for C4 B.C.'s

Similar small effects were found for lighter and heavier rings and also in the other studies of ring-stiffened shells [34] and ring-stiffened corrugated cylinders ($\rho_{PRE} \simeq 1.0$ in Table 2 of [36]), as well as for fiber-wound-cylinders ($\rho_{PRE} = 0.96$-0.99 for C4 B.C.'s in Table 3 of [37]).

For stringer-stiffened shells, the influence of prebuckling deformations has been found to be similar to that in isotropic shells for light stiffening ($\rho_{PRE} \simeq 0.9$ for C4 B.C.'s for lightly stiffened shells in Table 2 of [37]). With heavier stiffening, the effect is either similar but smaller ($\rho_{PRE} =$ $= 0.96$-1.0 for the heavier shells in Table 2 of [37]), or differs noticeably, exhibiting significant increases in the buckling loads when prebuckling is considered. For example, a typical stiffened shell tested in [38], shell No.3, with $(L/R)=2.49$, $(R/h)=347$ and hence $Z=2052$, $(A_1/b_1h)=1.07$, $(I_{11}/b_1h^3)=$ $=10.8$, $(e_1/h)=-6.0$, $\eta_{t1}=18.6$, yielded in [37]

$\rho_{PRE} = 1.053$ for SS3 B.C.'s

$\rho_{PRE} = 0.995$ for C4 B.C.'s

Almroth and Bushnell [37] also computed a short moderately stiffened shell, with $(L/R)=0.48$, $(R/h)=1215$ and hence $Z=267$, $(A_1/b_1h)=0.364$ but large eccentricity $(e_1/h)=\pm 5.66$, yielding for external stringers $(e_1/h)=-5.66$

$\rho_{PRE} = 1.61$ for SS3 B.C.'s

$\rho_{PRE} = 1.59$ for C4 B.C.'s

and for internal stringers $(e_1/h)=5.66$

$\rho_{PRE} = 0.822$ for SS3 B.C.'s

$\rho_{PRE} = 1.24$ for C4 B.C.'s

A typical medium length shell tested in [39] with heavier external stringers but smaller eccentricity, $(L/R)=1.01$ $(R/h)=513$ and $Z=501$, $(A_1/b_1h)=0.779$ and $(e_1/h)=-3.76$, shows a much smaller effect (see Table 1 of [39]):

$\rho_{PRE} = 1.03$ for SS3 B.C.'s

and

$\rho_{PRE} = 0.937$ for SS4 B.C.'s

From these and similar results in [36], it is evident that in stringer-stiffened shells under axial compression the effect of prebuckling deformations depends strongly both on the shell geometry parameter and the stiffener geometry parameters (A_1/b_1h) and (e_1/h). As, however, only a few unrelated cases have been studied until now, a partial parametric study is carried out with the BOSOR 3 program [40] and the linear theory of [43] in order to isolate the primary trends.

Figure 1 presents the variation of the influence of prebuckling deformation on the buckling loads with shell geometry for two types of stringer-stiffened shells. One with weak external stringers $(A_1/b_1h)=0.25$,

$(I_{11}/b_1h^3)=2.5$, $(e_1/h)=-2.5$ and $\eta_{t1}=2.5$, and one with medium external stringers $(A_1/b_1h)=0.5$, $(I_{11}/b_1h^3)=5$, $(e_1/h)=-5$ and $\eta_{t1}=5$. The curves of ρ_{PRE} are presented for SS3 and SS4 boundary conditions. Curves for isotropic sheils with the same boundary conditions, reproduced from [27] are also presented for comparison. The general trends appear to be: (1) a shift to higher Z with increase in stiffening; (2) a large increase in buckling loads for short shells, again more pronounced the heavier the stringers; (3) even for long shells a smaller decrease in ρ_{PRE} than in isotropic shells, and (4) a reversal of the relative magnitudes of ρ_{PRE} with SS3 and SS4 B.C.'s compared to those for isotropic shells.

The variation of ρ_{PRE} with stringer geometry for 3 typical shell geometries is plotted in Fig.2. (A_1/b_1h) is taken as the representative stringer geometry parameter, but the other parameters are also varied proportionally, the exact relation of the other parameters to (A_1/b_1h) being given in Table 2. Curves for both SS3 and SS4 B.C.'s are presented in the

Table 2: Stringer Geometry Parameters for Calculation of Effects of Prebuckling Deformations and Axial Restraint.

A_1/b_1h	0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.75	0.80	1.0
I_{11}/b_1h^3	0.5	1.0	2.0	2.5	3.0	4.0	5.0	6.0	7.5	8.0	10.0
$-e_1/h$	1.0	1.5	2.0	2.5	3.0	4.0	5.0	5.0	5.0	5.0	5.0
η_{t1}	0.5	1.0	2.0	2.5	3.0	4.0	5.0	6.5	10.0	12.0	20.0

figure. The large increase in buckling loads for short shells is immediately evident from the curves for Z=238. One can also observe the initial large rate of growth in ρ_{PRE} with stringer geometry up to $(A_1/bh) \approx 0.5$. The growth in ρ_{PRE} then subsides and ρ_{PRE} even tends to decrease slightly at $(A_1/b_1h) > 0.75$ for the stringer geometries given in Table 2. This behavior persists also at large Z where $\rho_{PRE} \approx 1$.

One can summarize the salient features of this partial parametric study for stringer-stiffened shells, and of the cases studied by other investigators, in the following practical conclusions:

1) For short shells with medium or heavy external stringers ($Z < 250$-400 and $(A_1/b_1h) > 0.3$ with corresponding other geometry parameters, in particular - $(e_1/h) > 3$), consideration of nonlinear prebuckling deformations results in substantial increases in buckling loads. Hence linear theory will yield very conservative predictions for such shells. For similar internal stringers this increase disappears and ρ_{PRE} has values close to those of isotropic shells. Further parametric studies for internally stiffened shells are needed.

2) For medium and long shells ($Z > 1000$) with external stringers, $\rho_{PRE} \approx 0.95$-1.05 and the effect of nonlinear prebuckling deformations may therefore be neglected.

3) For external stringers, ρ_{PRE} for SS4 is practically always below the value for SS3 B.C.'s, whereas in isotropic shells ρ_{PRE} for SS4 always exceeds that for SS3.

Thus for design purposes, one can safely neglect non-linear prebuckling deformations for externally stringer-stiffened cylindrical shells under axial compression, except in the case of very short shells, where predictions would be unduly conservative .

The influence of nonlinear prebuckling deformation on the vibrations of axially loaded stringer-stiffened shells has been studied in [63], as the particular case of zero load eccentricity. The predictions of linear theory [53] for a typical shell have been compared with those obtained when non-linear prebuckling is considered by BOSOR 3 [40]. At zero load, practically no difference has been found between the two theories for SS3 boundary conditions, as well as for SS4 B.C.'s for small n (for $n>6$, a very small difference appears). After the application of load, in the case of SS3 boundary conditions, the frequencies given by the linear theory are slightly lower for $n=3$ and 4 and slightly higher for $n>4$. At $n \geq 9$ the direction changes again and the linear theory yields the lower curves. These differences increase with load. For SS4 boundary conditions, the predictions with nonlinear prebuckling are always lower. The difference is greater than that in the SS3 case and becomes significant, though still relatively small, for higher loads. For example, for the vibration mode shown in Fig.10, $\rho_{PRE}=0.99$ for SS4 B.C.'s at zero load (where ρ_{PRE} is the ratio of frequency obtained with nonlinear prebuckling to that obtained with linear theory), and decreases to $\rho_{PRE}=0.93$ near buckling. The effect for SS3 is even smaller.

3. EFFECT OF IN-PLANE BOUNDARY CONDITIONS

The influence of in-plane boundary conditions on buckling under axial compression, which has been found to be very significant for isotropic shells([1]-[5] and other studies), proves to be even more important in stiffened shells. Earlier studies, [9],[35]-[38], considered some of the different in-plane boundary conditions. Preliminary parametric studies were carried out in [41], but more complete ones have only recently been performed (see [32],[42] and [43]).

For ring-stiffened cylindrical shells under axial compression the effects are similar to those in isotropic shells - the "weak in shear" boundary conditions SS1 and SS2 (with $N_{x\phi}=0$ instead of $V=0$), that rarely occur in practice, reduce the buckling load to about half its classical value, whereas it is not affected by axial restraint ($u=0$ instead of $N_x=0$) and hence SS3 and SS4 loads are practically identical, (see Figs.2 and 3 of [42]). Clamped edges have not yet been fully studied, but partial results indicate similar trends. Heavy internal rings exhibit the only significant deviation that the weakening effect of the "weak in shear" boundaries tends to disappear (Fig.5 of [42]), without changing the negligibity of the effect of axial restraints.

For stringer-stiffened cylindrical shells under axial compression the influence of in-plane boundary conditions differs appreciably from that in isotropic shells - the main difference being that the axial restraint ($u=0$ instead of $N_x=0$), which has no effect in unstiffened shells under axial compression [2] or [44], becomes the predominant factor, whereas the circumferential restraint ($v=0$ instead of $N_{x\phi}=0$) has only a minor influence. Hence, for heavy or medium stringers, SS3 and SS1 boundary conditions yield practically identical results and similarly SS1 and SS2 hardly differ (see for example Figs.2, 4 or 5 of [43], or Fig.6 of [32]). The studies of [43] show that the effects also strongly depend on the shell geometry parameter Z and on the stringer parameters (A_1/b_1h) and (e_1/h). The effect of axial restraint is more pronounced for internal stringers, but even for external ones axial restraint ($u=0$, SS2 or SS4) may raise the predicted buckling load for medium or heavily stiffened shells by 50% or more if the shell is long. For weak stringers, the relative importance of axial and circumferential restraints again approaches the familiar one of isotropic shells, though even weakly stiffened shells still differ considerably (see Fig.3 of [43]). The effect of axial restraints are much smaller for stringer-stiffened shells with clamped ends. For example, in shells AS2 and AS3 tested in [35], $(P_{C3}/P_{C4}) = 1.003$ and 1.002, respectively,(whereas with simple supports $(P_{SS4}/P_{SS3}) = 1.37$ and 1.14 respectively, where P_{C3}, P_{C4}, P_{SS3}, P_{SS4} are the theoretical buckling loads for the corresponding boundary conditions.

For better assessment of the test results, an additional parametric study of the effect of axial constraint on the buckling under axial compression of simply supported stringer-stiffened shells in the relevant geometry range has been carried out with the theory of [43] and with BOSOR 3 [40], taking into account nonlinear prebuckling in the latter. Figure 3 shows the variation of the stiffening, afforded by axial restraint, with stiffener geometry for three values of Z. The stringers are external and the relation of the other stringer geometry parameters to the area ratio (A_1/b_1h) is given in Table 2. The predictions of Fig.3 indicate that, for the geometries studied, which include the range of the test shells, the largest effect occurs for medium stiffening, falling off as the stiffening increases further. By correlation with Figs.4 and 5 of [43], one can explain this behavior as resulting from the shift to higher Z of the maximum of the axial constraint effect with increase in stringer area and other geometry parameters. Hence for a given Z the increase in (A_1/b_1h) yields the apparent decrease in (P_{SS4}/P_{SS3}).

One may note that the influence of axial contraints on the buckling of axially compressed stringer stiffened-shells resembles that observed in isotropic shells buckling under lateral or hydrostatic pressure (see for example [45] or [46]) or vibrating freely ([47] or [48]). The similarity with the influence of in-plane boundary conditions for vibrations is encouraging if correlation between vibration and buckling tests is being attempted.

In the studies of vibrations of stiffened shells the influence of in-plane boundary conditions has apparently not been considered. The investigators have limited themselves to consideration of rotational restraints and have analysed only SS3 or C4 boundary conditions (see for example [49]-[52]), though reference is sometimes made to the importance of axial restraints in isotropic shells [49]. Hence considerable attention was given to the influence of in-plane boundary conditions in the theoretical and experimental studies of vibrations of axially loaded cylinders at the Technion. The earlier tests and calculations ([53] and [54]) considered only clamped edges C4. Then supports without rotational restraints, similar to those developed for buckling tests ([55] or [56]), were employed in tests [57] and [58], and concurrently extensive computations were carried out for different in-plane boundary conditions. As a result of the prominence of axial constraints in the case of buckling, the emphasis in the vibration studies has also been on axial constraints (see [58]).

4. OUT-OF-PLANE BOUNDARY CONDITIONS

Concurrently with the many investigations of the effect of in-plane boundary conditions on the buckling of cylindrical shells, the out-of-plane boundary conditions were also re-examined (see, for example, [1],[59] or [60]). These studies on isotropic shells indicated that free edges cause very significant reductions in buckling loads also under axial compression. For elastic out-of-plane boundary conditions two types of buckling modes have to be considered: one similar to the usual mode with supported edges and one an inextensional (n=2) buckling ode which can occur for weak radial edge restraints or weak supporting rings and which yields very low buckling loads [61]. If the radial elastic edge restraint is sufficient to prevent the inextensional mode, the buckling load will not be significantly below that of a simply supported cylindrical shell([60] or [61]). Since extremely weak edges only rarely occur in practice, the out-of-plane boundary conditions have not received attention in recent years and their effect on stiffened shells has apparently not been studied.

For vibrations of unstiffened shells, free ends are known to lower the minimum frequencies considerably (see for example [51]). An n=2 mode ("ovalling"), somewhat similar to the inextensional buckling mode predicted for free ends in [61], has also been observed prior to collapse in long shells with one end clamped [62]. In [49] and [51] the effect of different out-of-plane boundaries at one edge, on the vibrations of isotropic and stiffened shells, has been studied theoretically and experimentally. Good agreement was found between tests and theory. Stringer-stiffened cylinders tested showed minimum frequencies similar to those of the corresponding isotropic shells, and both decreased substantially (to less than half) when one end was left free or to very small values when both ends were free [51]. This decrease is accompanied by a corresponding shift to a lower n for minimum frequency. The related ring-stiffened shells had a minimum frequency about 50% higher than the isotropic ones with $C\ell$-$C\ell$ (C4-C4) boundary conditions, which decreased less (to about two thirds) when one end was free, and was accompanied also by a smaller shift to a lower n for minimum frequency. The ring-stiffened shell was, therefore, found to be less sensitive to a weakening of radial constraints than the corresponding isotropic or stringer-stiffened shell (stringers being very ineffective in raising the frequencies). However, rather weak rings were found to suffice for long shells, to restore the effect of a radially rigid support [62].

For buckling under axial compression the feasibility of the inextensional buckling-mode appears to be the primary factor governing possible significant reduction in buckling loads. When one extends the energy method analysis of [61] to stiffened shells, one finds that in stringer-stiffened shells practically only the torsional stiffness of the stringers is effective in the inextensional buckling mode. Hence their behavior does not differ much from that of corresponding isotropic shells. However, the radial restraints or end rings required to eliminate the inextensional mo . though larger than predicted in [60], will usually be provided by the supporting structure or laboratory fixtures.

If one is concerned about a more local out-of-plane freedom, as in the tests in [39] and [63], the n=2 inextensional mode is not applicable and the usual $n \geq 2$ buckling modes should be considered. Then the original conclusions of [60], that even small radial restraints are practically equivalent to w=0, should hold. Some preliminary calculations for the stringer-stiffened shells of [63] confirm this both for buckling and vibrations.

5. EXPERIMENTAL DEFINITION OF BOUNDARY CONDITIONS BY CORRELATION WITH VIBRATION TESTS.

The dominant influence of the boundary conditions on the buckling of stringer-stiffened cylindrical shells emphasizes the importance of their exact definition. The similarity with the corresponding influence on the vibrations of such shells, in particular for the lower natural frequencies indicates that correlation with vibration tests may be an appropriate tool to achieve this definition. For columns, many correlation studies have been developed for assessment of the elastic restraints provided by the boundaries from vibration tests (see [64]-[68]). It was pointed out in [66] and [68] that the success of these techniques is restricted to supports with zero transverse displacements or strong transverse springs. Though analogous methods have apparently not been developed for shells, the similarity in behavior observed in the vibration and buckling tests of stiffened shells lead the authors to believe that such techniques could here be successfully employed.

Furthermore, if the effect of the boundary conditions on the buckling loads of stiffened shells is as important as that of the initial imperfections, or even predominant, their proper consideration should reduce experimental scatter significantly. The ratio of the experimental buckling load P_{EXP} to that predicted by linear theory P_{CR}, called in [55] and subsequently "linearity" $\rho=(P_{EXP}/P_{CR})$, is usually not far from unity in closely stiffened shells (see [32]). However, there is still a scatter of approximately ±20% in the collected experimental results (see Figs.8 and 9 of [32] or Figs.12 and 19 of [56]). This scatter can indeed be reduced by correlation with vibration tests.

Two typical shells tested recently [57], RO-33 and RO-34, are now examined as an example of the correlation technique developed.

The dimensions of the specimens are given in Table 3. The shells were manufactured as twins from one blank and on one mandrel (see [56]) and are considered as twins, though their dimensions differ very slightly. The main difference between the two specimens was in their boundary conditions. Shell RO-33 was clamped, approaching the theoretical C4 B.C.'s. Clamping for RO-33 is affected in the usual manner (see also [54]): the end plates

Geometrical Property \ Shell	RO-25	RO-26	RO-27	RO-28	RO-29	RO-30	RO-31	RO-32	RO-33	RO-34
Radius to shell middle surface, R(mm)	120.1	120.1	120.1	120.1	120.1	120.1	120.1	120.1	120.1	120.1
Shell Thickness, h(mm)	0.253	0.251	0.254	0.254	0.255	0.259	0.249	0.257	0.251	0.245
Shell Length, L(mm)	130.0	130.0	130.0	130.0	130.0	130.0	215.0	120.0	130.0	130.0
Stiffener Width c_1(mm)	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.90
Stiffener Height d_1(mm)	1.505	1.488	1.480	0.979	0.974	0.980	1.985	1.984	1.745	1.749
Number of Stiffeners	84	84	84	84	84	84	84	84	84	84
R/h	475	479	473	473	471	463	482	467	479	490
L/R	1.08	1.08	1.08	1.08	1.08	1.08	1.79	0.999	1.08	1.08
Z	530	535	528	528	526	518	1474	445	534	548
A_1/b_1h	0.596	0.594	0.584	0.386	0.383	0.379	0.799	0.775	0.698	0.716
$-e_1/h$	3.47	3.47	3.41	2.43	2.41	2.39	4.48	4.35	3.97	4.06
I_{11}/b_1h^3	0.776	0.785	0.754	0.499	0.490	0.471	4.236	3.847	2.809	3.042
η_{t1}	6.598	6.678	6.375	3.099	3.025	2.943	10.450	9.495	8.513	9.175

7075 Aluminium Alloy $E = 7500\ kg/mm^2$, $\nu=0.3$, Specific Gravity = 2.80 .

Table 3: Geometrical Properties of Stringer-Stiffened Shells Tested.

have circular grooves, the inner diameters of which fit the test shell tightly . Cerrobend (a low melting metal) is then poured into the external gaps. The depth of the groove was 10 mm, and the length of shell RO-33 was therefore originally 20 mm longer than RO-34. Hence the specimens were finally of identical length. Shell RO-34 was mounted on simple suports , shown as Edge A in Fig. 9. With these B.C.'s the shell rests in a triangular groove which is carefully fitted to the radius of each shell. The buckling loads predicted with linear theory (taking into account the small dimensional differences), the experimental buckling loads and the "linearity" values are:

<u>RO-33</u> $P_{CR} = 6755$ kg <u>C4</u>

$P_{EXP} = 4300$ kg

$\rho = 0.64$

<u>RO-34</u> $P_{CR} = 3644$ kg <u>SS3</u>

$P_{EXP} = 4210$ kg

$\rho = 1.16$

Hence there is an apparent scatter of about 45 percent. The experimental boundary conditions are, however, somewhere between SS3 and SS4 in the case of RO-34, and also not complete clamping for RO-33, but less than C4.

The influence of axial elastic restraint k_1 and rotational restraint k_4 on buckling and vibrations can be calculated with the theory of [53], extended to include elastic restraints at the boundaries [58]. Fig.4 shows the variation of buckling load between SS3 and SS4 and between SS4 and C4 boundary conditions for these two shells. The variation between SS3 and SS4 B.C.'s is achieved by introduction of an axial spring k_1 (the other B.C.'s are $v=0$, $w=0$, $M_x=0$). The stiffness k_1 is zero for SS3 B.C.'s and infinity for SS4. Similarly, the variation between SS4 and C4 is obtained with a torsional spring (the other B.C.'s are $u=0$, $v=0$, $w=0$), the stiffness of which k_4 is zero for SS4 and infinity for C4 B.C.'s . The variation of buckling load with increasing spring stiffness shows in both cases initially a steep increase and then a slow asymptotic approach to the values of SS4 and C4. It should be pointed out that the magnitudes of the springs in the two cases are different. One may also note, that in the figure the small dimensional difference in shell thickness between the two shells is taken into account and the precise results are presented for both shells. The buckling modes in all cases have here one axial half wave, whereas the circumferential wave numbers for small values of axial springs $(0 < k_1 < 4)$ is $n=10$, and for all other cases $n=11$.

Correlation with the natural vibrations of the corresponding axially loaded shell is now employed to assess the real boundary conditions of the experiment. In Fig.5 the influence of elastic axial and rotational restraints on the squared frequency of the mode $n=11$, $m=1$, at an axial load of 2000kg, is shown in the same manner in which the influence on buckling load was shown in Fig.4. This mode of vibration was chosen, because it represents the buckling mode in most of the range of the springs. The load of 2000 kg was chosen.to be well outside the range of low loads at which the shell has not yet settled in its simple supports. The behavior is indeed very similar to that shown in Fig.4. Hence by measuring the natural frequency at a relatively low load, one can estimate the real boundary conditions.

The experimental technique developed for vibration tests of axially loaded shells [54] is employed here. The test apparatus and procedure is described in detail in [54] and is only briefly summarized here. The test set up is shown in Fig.6. The shell is excited by an accoustic driver which is inside the shell. The reponse of the shell is measured by a microphone outside the shell. The excitation frequency is changed and resonance is detected by the help of Lissajous figures. When a resonance frequency is detected, the mode of vibration is recorded by plotting the microphone reading versus its circumferential or axial position on X-Y recorders. The load is applied with a screw-jack and the load distribution is checked by an array of pairs of strain gages, which permit separation between compressive and bending strains. In the tests, the load is increased, first in relatively large increments of 400kg, and then at progressively smaller increments of 200kg and 100kg. At each load, a full scan of a wide range of frequencies is performed and the modes at the resonant frequencies are recorded. The scan is repeated with the microphone at different positions to prevent ommission of any mode due to an accidental node-point. At buckling the drop in load is also recorded. The experimental results include some multiple results for the same mode shape at a certain load. These appear because of difficulties in detection, and similar ambiguities were attributed by Tobias [60] to imperfect axisymmetry. The multiple frequencies, however, do not cause any difficulty when they are close, as in the case here.

Figure 7 presents the frequency squared versus axial load in shells RO-33 and RO-34 for the mode n=11, m=1. For clarity, the theoretical curves are presented only for one shell, RO-34, since the results for the twin shell RO-33 are very close (as can be seen for example in Fig. 5).

The experimental results for RO-34 exhibit a clear trend to some value of the spring k_1 and those for RO-33 to a certain value of the spring k_4. . ig.8 is a similar plot for another mode n=7, m=1, and again shows similar clear trends. The values of k_1 and k_4 from Fig.8 are, however, found to be slightly lower than those in Fig.7 for n=11, m=1. Similar variations in the restraints with wave number were also observed in other cases,(see[58]).

The discussion will now focus on the case n=11, m=1, since this is also the theoretical buckling mode. The values of frequency squared at 2000kg are taken from Fig.7 and by following the dotted lines in Fig.5, one finds that for RO-33, k_4=1660 and for RO-34, k_1=20. With these values of spring stiffness one turns to Fig.4 and, by again following the dotted lines, one obtains the theoretical predicted buckling loads for these values of springs. Hence the predictions and "linearity" values for the same experimental buckling loads change to:

<u>RO-33</u> P_{CR} = 6469 kg. elastic restraints(from vibrations)

P_{EXP} = 4300 kg.

ρ = 0.66

<u>RO-34</u> P_{CR} = 5288 kg. elastic restraints(from vibrations)

P_{EXP} = 4210 kg.

ρ = 0.80

The scatter in the "linearity" reduces therefore appreciably from the previous 45% to 17%.

A similar correlation has been carried out for two other shells tested, RO-31 and RO-32. These two shells were again manufactured from one blank and on one mandrel and are twins except for different length. Their dimensions are given in Table 3. Shell RO-31 was clamped in the same manner as RO-33 and RO-32 was again supported as Edge A in Fig. 9. The results of the correlation with vibrations and the reference values are:

<u>RO-31</u> P_{CR} = 6870 kg. <u>C4</u> P_{CR} = 6250 kg. elastic restraints (from vibrations).

P_{EXP} = 4693 kg.

ρ = 0.68 ρ = 0.72 after correlation

<u>RO-32</u> P_{CR} = 5338 kg. <u>SS3</u> P_{CR} = 6220 kg. elastic restraints (from vibrations).

P_{EXP} = 4700 kg.

ρ = 0.88 ρ = 0.76 after correlation

Hence the scatter in "linearity" has been reduced from 23% to 5%.

Further correlations for other experimental results and further tests are under way to add confidence in the correlation technique.

6. EFFECT OF ECCENTRICITY OF LOADING

Eccentricity of loading, usually defined as the radial distance between the line of axial load application and the shell midskin, has been shown to have considerable influence on the buckling load of stringer-stiffened shells (see, for example, [36], or [39]). In the more recent study [39] the theoretical investigation was amplified by tests on integrally stringer-stiffened cylindrical shells loaded eccentrically and having different boundary conditions. These studies have now been extended to consider the influence of load eccentricity on the vibrations of axially loaded stiffened shells both theoretically and experimentally and correlate the results with those for buckling. Two families of shells are studied, one heavily stiffened and the other moderately stiffened.

The details of the extensive calculations and experiments are given in [63] and only the salient features are summarized here.

The experimental set-up and procedure is essentially identical to that shown in Fig.6 and discussed in the previous section, (see also [54] or [58]), except for the details of load application and the specimens.

Six integrally stringer-stiffened shells were tested in the present test series. The specimens, which are similar to the shells of [54] and [56], where cut from 7075-T6 aluminium alloy extruded tubes and accurately machined by a process described in [56]. The eccentricity of loading is achieved by applying the load through the stringers. Specimens are therefore manufactured with three kinds of edges, as shown in Fig.9. In the case of edge A, load is applied through the mid-skin of the shell, for edge B load is applied through an intermediate point along the depth of the stringers and in the case of edge C through the tip of the stringers. In all the cases special support rings (see Fig .9) are accurately fitted to the shell edges, which restrain the radial displacement of the shell edge or stringers. The specimens were manufactured in triplets, consisting of three shells made from one blank, one with each of the 3 types of edges. Comparison was therefore between almost identical shells, as can be seen in Table 3, giving the dimensions of the shells. Specimens RO-25,26 and 27 represent the heavily stiffened family of shells and RO-28,29 and 30 the moderately stiffened one.

The calculations for eccentric loading were carried out with BOSOR 3 [40] and for $\bar{e}=0$ they were compared with results obtained with the linear theory of [53]. Vibrations with one axial half wave (m=1) and two axial half waves (m=2), and loads up to buckling, are computed and measured. Results for a typical vibration mode (m=1, n=7) are shown in Fig.10. Theoretical predictions are given for SS3 and SS4 boundary conditions and the experimental results for the 3 shells of the family are also plotted. The experimental results exhibit here a behavior similar to that predicted for SS4 B.C.'s(through the values differ): RO-25 with $(\bar{e}/h) = 0$ having the lowest frequencies and those corresponding to RO-27, with $(\bar{e}/h)=-2.84$, and RO-26 with $(\bar{e}/h) = -6.43$ above them respectively. This is typical for high circumferential wave numbers ($n \geq 7$ for the shells with heavy stringers and $n \geq 8$ for those with medium stringers), whereas for lower n the experimental behavior is closer to that predicted for SS3 B.C.'s and even the frequencies are below the ones for SS3 (see ([63]).

The influence of eccentricity of loading on the buckling loads is shown in Figs.11 and 12. The theoretical predictions for the heavily stiffened shell (Fig.11) show that for SS3 B.C.'s there is little effect up to an outward load eccentricity of $(\bar{e}/h)=-2.5$, where a shallow maximum occurs. For larger outward load eccentricities the buckling load decreases and also the mode of buckling changes. For SS4 B.C.'s there is initially a steep rise in the buckling load with increase of outward eccentricity up to $(\bar{e}/h)=-1.5$. After that the increase is more moderate. The case of SS4 boundary conditions with load through midskin plus an equivalent moment is also plotted. This case differs from the former case of SS4 (with coinciding support and loading point) and is closer in its behavior to the SS3 case. We see that the theoretical influence of load eccentricity on the buckling load is similar to its influence on vibrations, especially for SS4 B.C.'s where also for vibrations there is a steep rise in frequencies for small outward load eccentricity and a more moderate one for higher eccentricities (see for example, Fig.5 j of [63]). The three experimental points are also plotted in Fig.11. The three shells buckled with m=1, but exhibited different behavior near buckling. RO-25 buckled at 3700kg. with 8 circumferential waves and the load dropped after buckling to 1600kg. Shell RO-26 showed noticeable bending before buckling. Buckling occured at 1990kg. (it was not so well defined and the number of waves could not be caounted). After buckling the load dropped only to 1960kg, the shell then continued to carry higher loads and the waves developed with increasing load (see also Figs.9a and 9b of [63]). At 2100kg and 2150kg (the maximum load the shell carried), there were approximately 12 waves. Shell RO-27 buckled at 2500kg with approximately 10 waves and after buckling the load dropped to 2240kg. Note that for buckling calculations the load eccentricity measured after buckling is employed $(\bar{e}/h) = -5.66$ for RO-26 and $(\bar{e}/h) = -2.04$ for RO-27.

The results for the moderately stiffened shells are shown in Fig.12. Though generally similar, Fig.12 exhibits some significant differences compared to Fig.11. The theoretical curve for SS4 differs having a steep maximum at a relatively low value of load eccentricity and decreasing then with $(\bar{e}/h)$. This behavior correlates with a similar one for the vibrations (see [63]). The experimental buckling loads were for RO-28: 2960kg.with 9 waves, for RO-30: 2100kg.with 9 waves and for RO-29: 1740kg.with 9 waves. The buckling behavior was again more violent for RO-28 and softer for RO-30 and RO-29.

Hence the present experimental results reconfirm clearly and emphasize the behavior observed in [39] that increase in outward load eccentricity results in lower buckling loads but also in a much less violent buckling phenomenon. The agreement between the experimental results and the predictions is fair, but might possibly be improved by correlation with the vibration results. One may observe from the vibrations tests, that the test boundary conditions are nearer to SS4 than SS3, in the vibration modes close to the buckling modes. Further studies and tests are planned.

7. PREDICTION OF BUCKLING LOADS FROM VIBRATION TESTS

The prediction of the buckling loads from vibration tests, as a basis for a nondestructive test method, has been attempted by many investigators for different structures. For columns and frames, good results have been obtained (see for example [70],[71] or [68]), but applications of similar techniques to plates and shells have not yielded practical methods ([70] and [11]), except one successful application to spherical caps [72]. Other methods of nondestructive testing have been developed for shells with some success (see for example [73] and [74]). The authors believe, however, that correlation with vibration tests is a promising direction of attack, in particular for closely stiffened shells, where the low vibration modes observed in tests are very similar to the buckling modes.

The prediction of buckling loads from the curves of frequency squared versus loads was already attempted by the authors in [54]. Some of the shells tested there yielded promising results, though they were relatively weakly stiffened. The fact, that in the buckling modes for the longer shells tested in [54] $m \geq 2$, also made the correlation more difficult. Heavier stiffening and shorter shells appeared therefore desirable for more reliable correlation, on which the initial steps in the developments of a nondestructive test method could be based.

In Figs. 13 and 14 the measured frequencies to the second, third and fourth power are plotted versus axial load for shells RO-33 and RO-34, respectively. Straight lines were fitted (least square error fit) to these points, and their intersections with the abscissa represent possible predicted buckling loads. These preliminary results show that the extrapolations of f^3 and f^4 bracket the experimental buckling loads fairly well, with f^4 yielding conservative predictions. The optimum power of f for these semi-empirical lines apparently depends on the shell geometry and on the boundary conditions. Further studies along these lines of previous test results, as well as additional tests, are in progress in order to develop a reliable non-destructive test method for stiffened cylindrical shells. It should be remembered that the correlation with the vibrations of the actual shell, loaded up to say half its buckling load - on which the method is based - will ensure that the real boundary conditions and imperfections are accounted for in the predicted buckling load. The authors feel that the preliminary results are very encouraging.

8. CONCLUSIONS

The following general conclusions can be drawn from the results obtained:

(1) For externally stringer-stiffened cylindrical shells under axial compression, in the range of geometries studied, nonlinear prebuckling deformations have a relatively small effect on the buckling loads, except for very short shells where consideration of prebuckling deformation yields significantly higher buckling loads.

(2) Axial restraints, and also rotational restraints, strongly affect the buckling loads of stringer-stiffened shells. This effect depends on shell and stiffener geometry. Hence a better definition of the actual boundary conditions, in particular with respect to axial and rotational

restraint, will lead to more accurate predictions of buckling loads and will significantly reduce experimental scatter.

(3) Axial restraints and rotational restraints also strongly affect the free vibrations of stiffened shells. This analogical behavior may be utilized as a tool for the definition of the actual boundary conditions, as was demonstrated for typical stringer-stiffened shells.

(4) Eccentricity of loading has a very significant effect on the buckling load and behavior of stringer-stiffened shells, as was predicted by theory and verified by tests. The vibrations are affected in a similar manner by load eccentricity.

(5) Preliminary results indicate, that correlation with vi[illegible]on tests, at axial loads much below the buckling loads, may yie[illegible] basis for a nondestructive test method for practical determination of buckling loads of imperfect closely stiffened shells with real boundary conditions.

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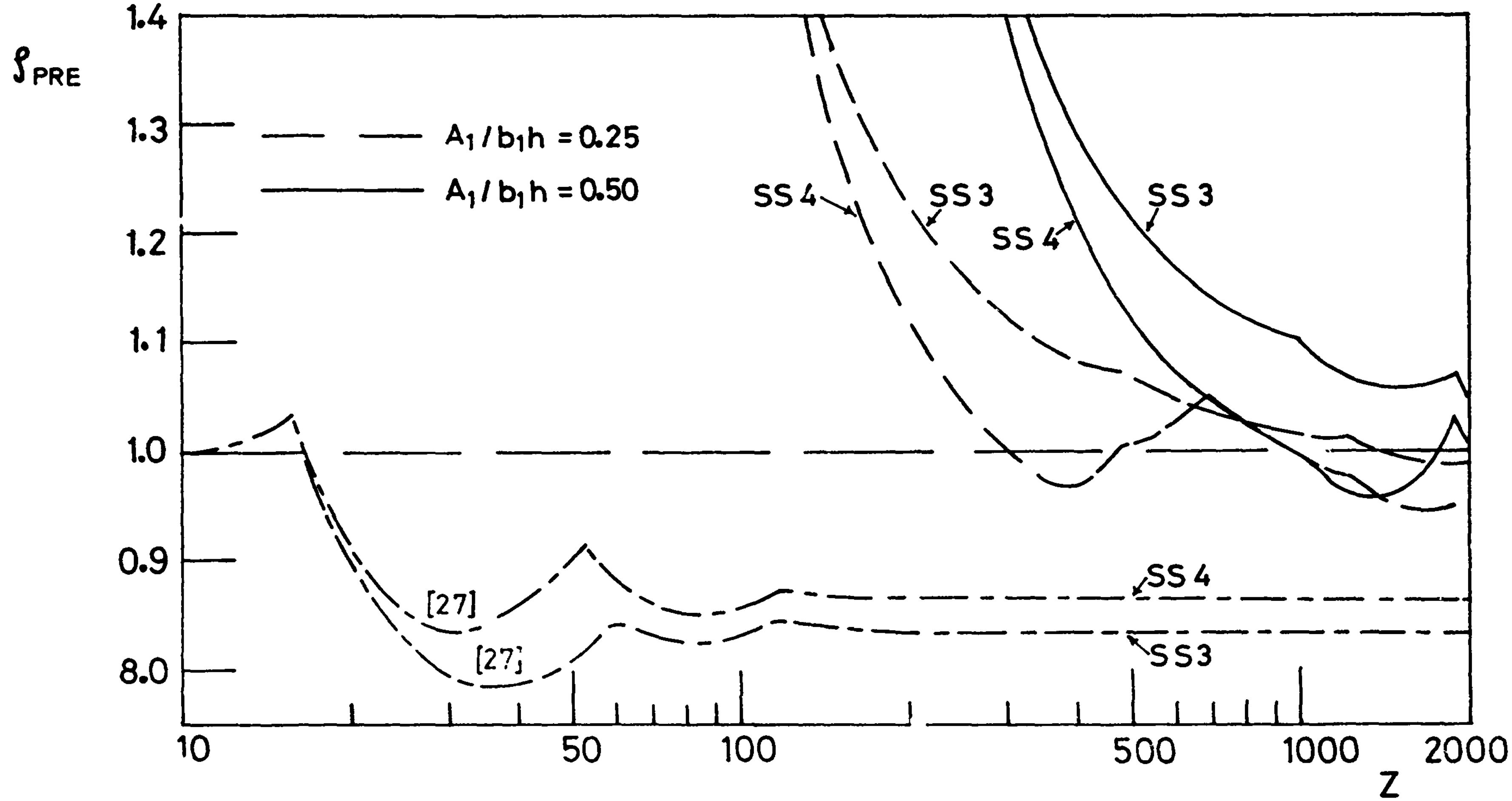

FIG. 1 EFFECT OF PREBUCKLING DEFORMATIONS ON BUCKLING OF STRINGER - STIFFENED SHELLS—VARIATION WITH SHELL GEOMETRY.

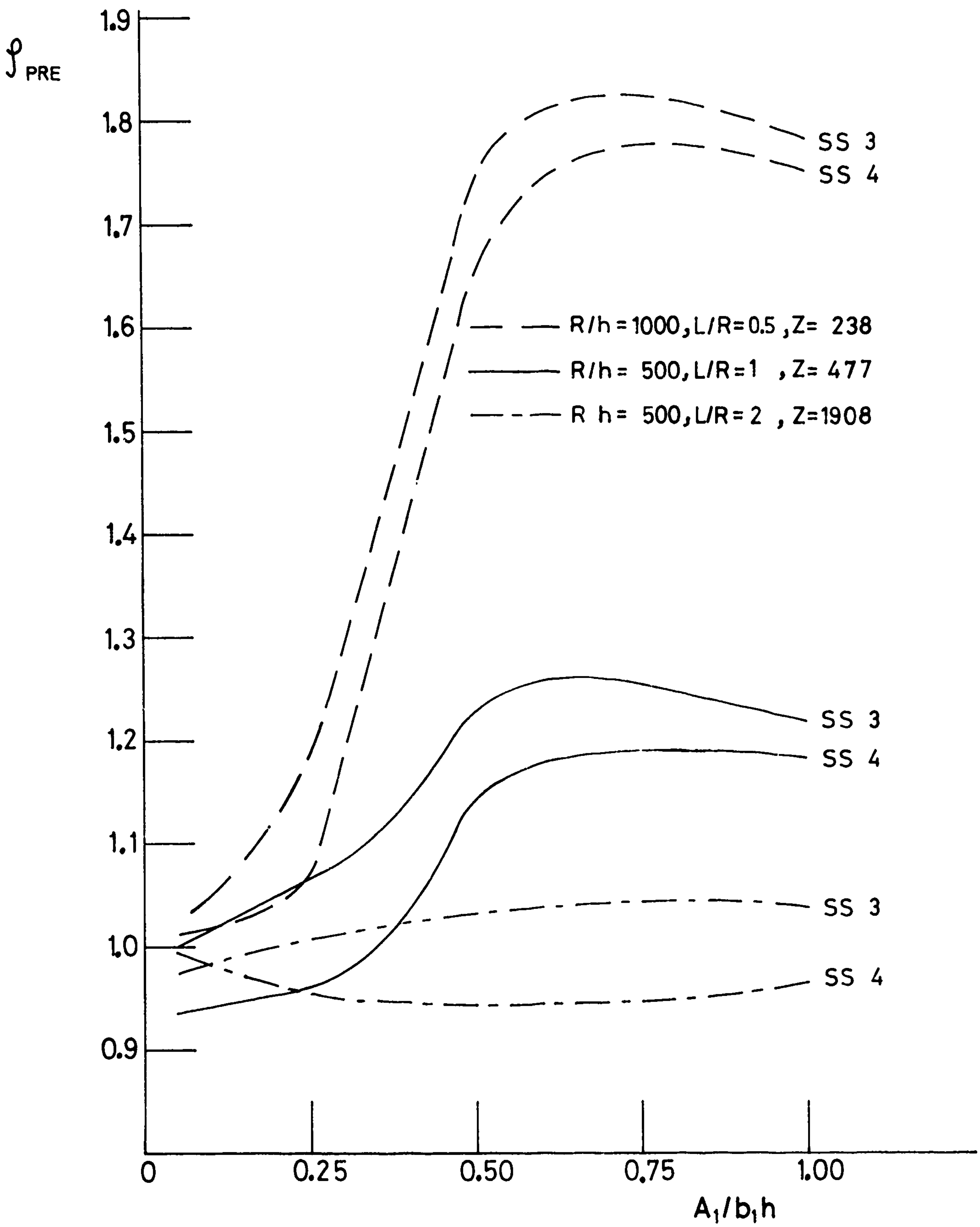

FIG. 2 EFFECT OF PREBUCKLING DEFORMATIONS ON BUCKLING OF STRINGER-STIFFENED SHELLS — VARIATION WITH STRINGER GEOMETRY.

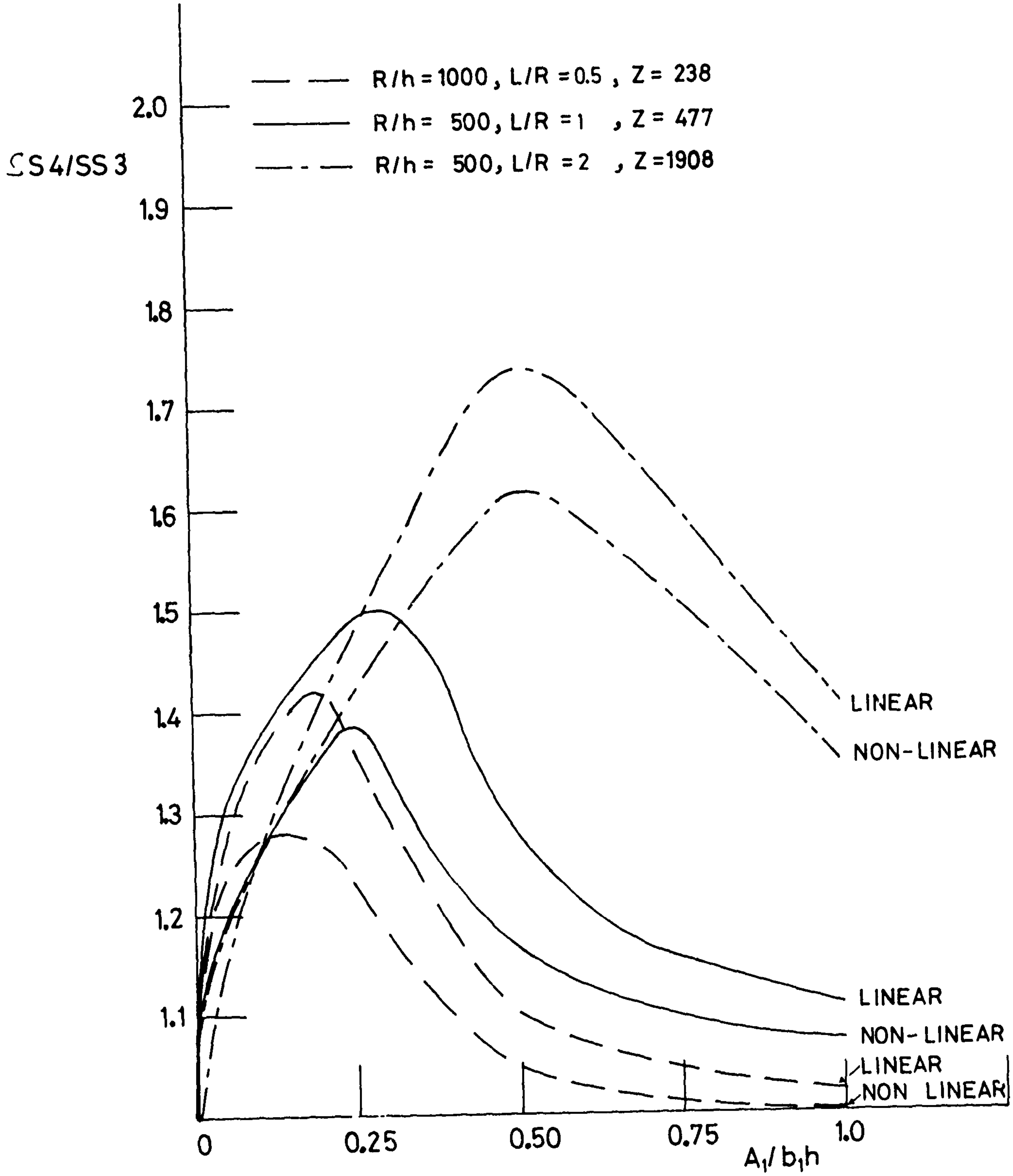

FIG. 3 EFFECT OF AXIAL RESTRAINT ON BUCKLING OF STRINGEF. – STIFFENED SHELLS.

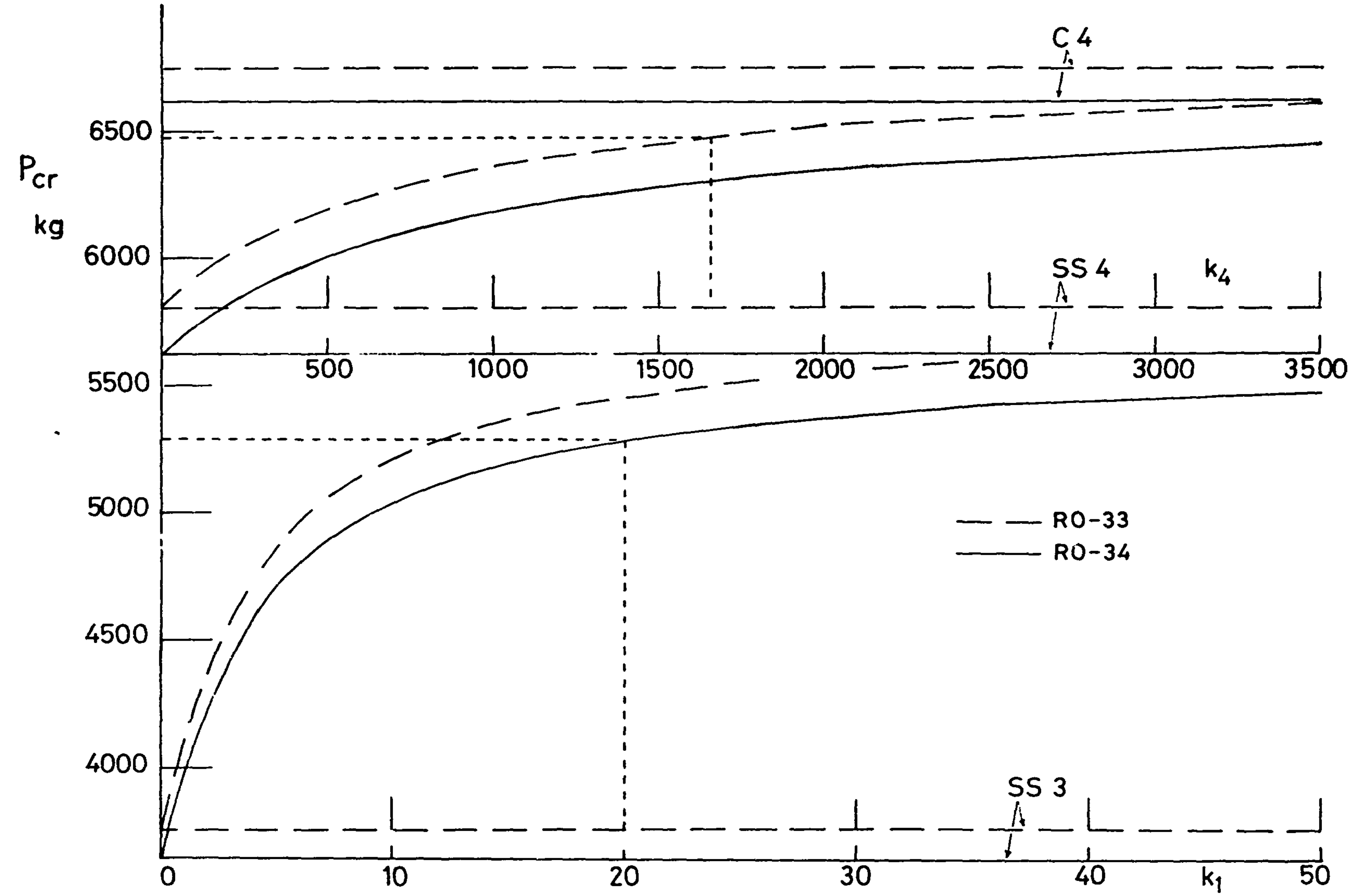

FIG. 4 INFLUENCE OF ELASTIC AXIAL AND ROTATIONAL RESTRAINT ON THE BUCKLING LOADS OF SHELLS RO-33 AND RO-34.

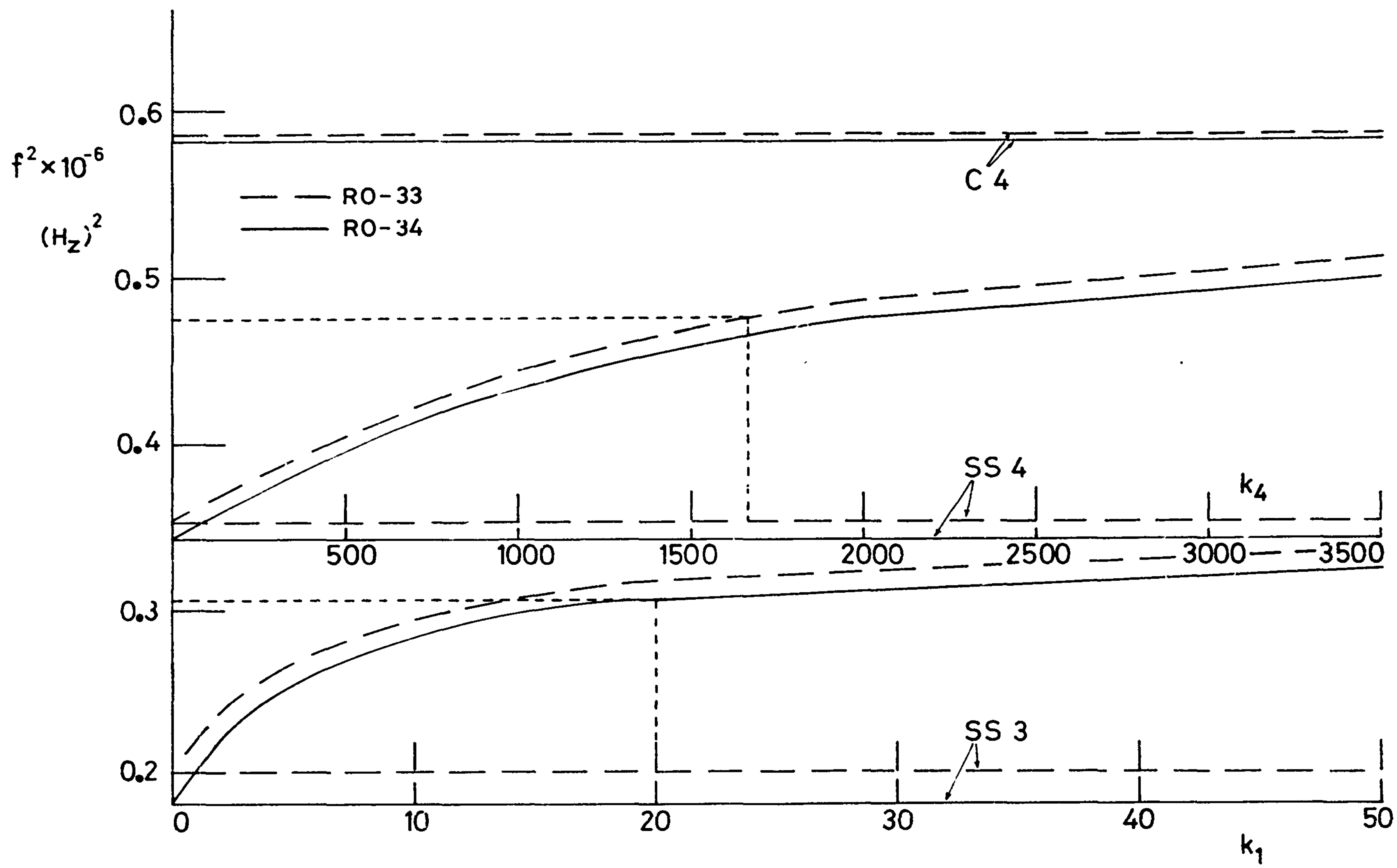

FIG. 5 INFLUENCE OF ELASTIC AXIAL AND ROTATIONAL RESTRAINT ON THE VIBRATIONS OF SHELLS RO-33 AND RO-34 (P = 2000 kg, n = 11, m = 1)

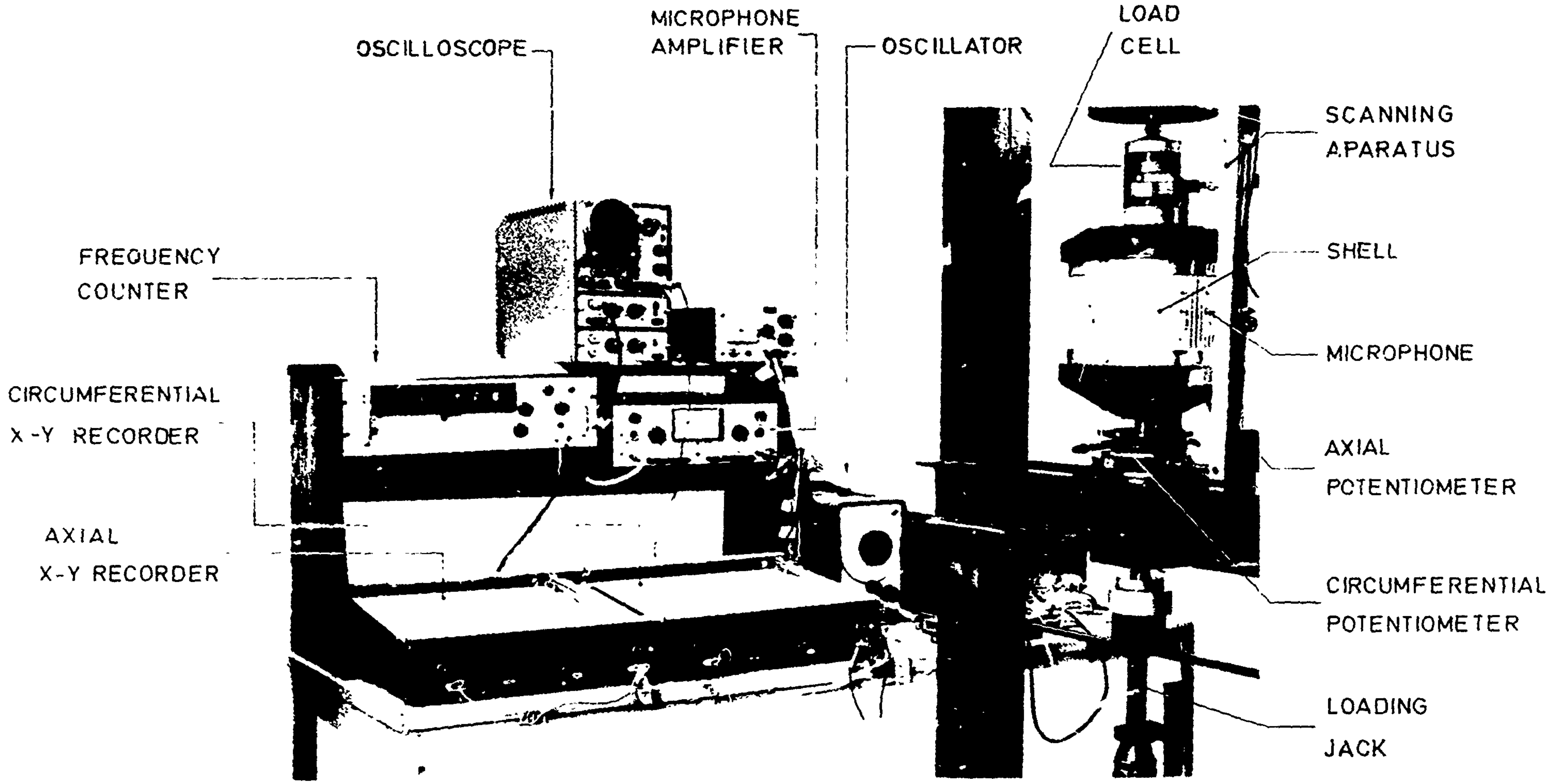

FIG. 6 TEST SET UP

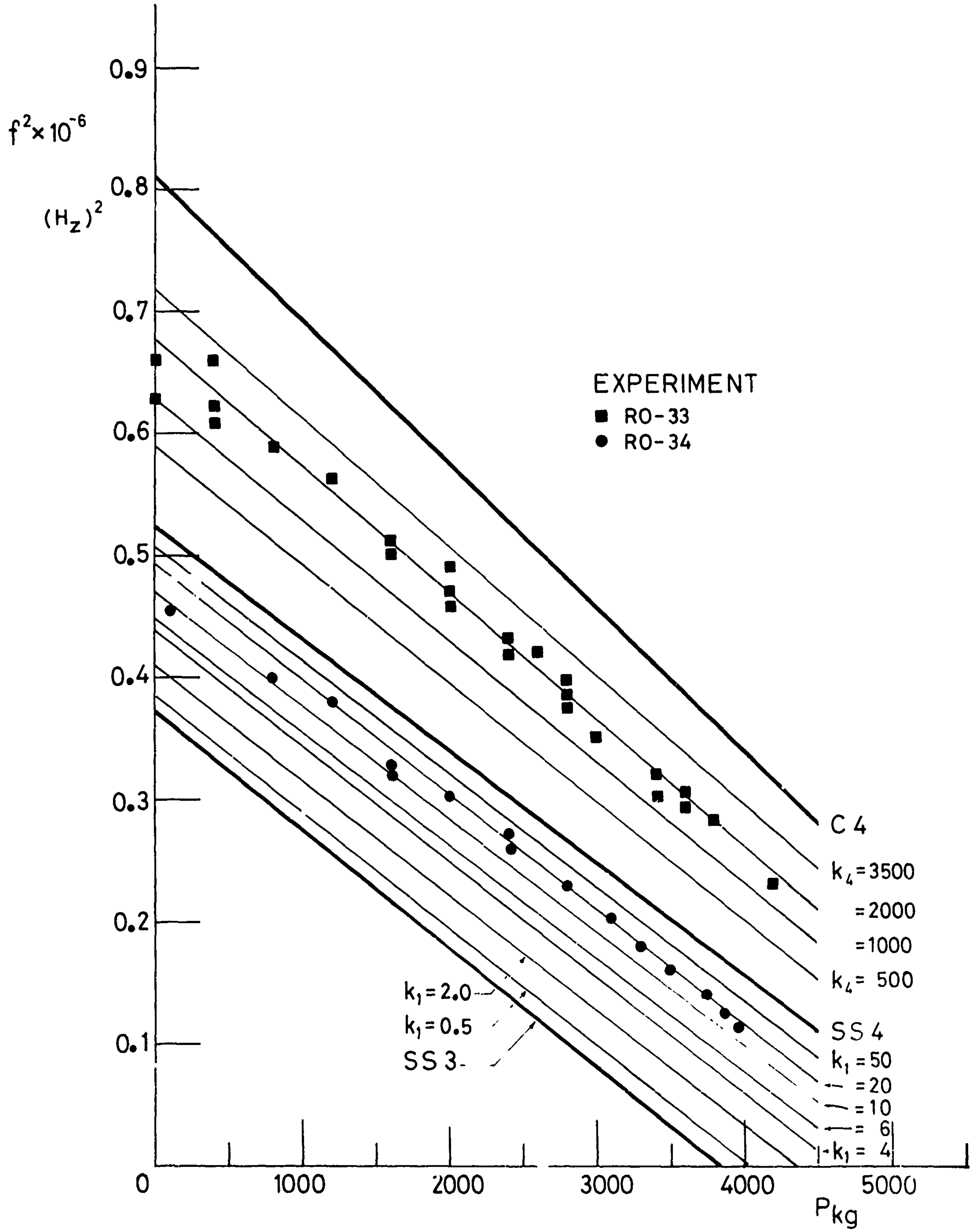

FIG. 7 FREQUENCY SQUARED VERSUS AXIAL LOAD-SHELLS RO-33 AND RO-34 FOR n=11, m=1.

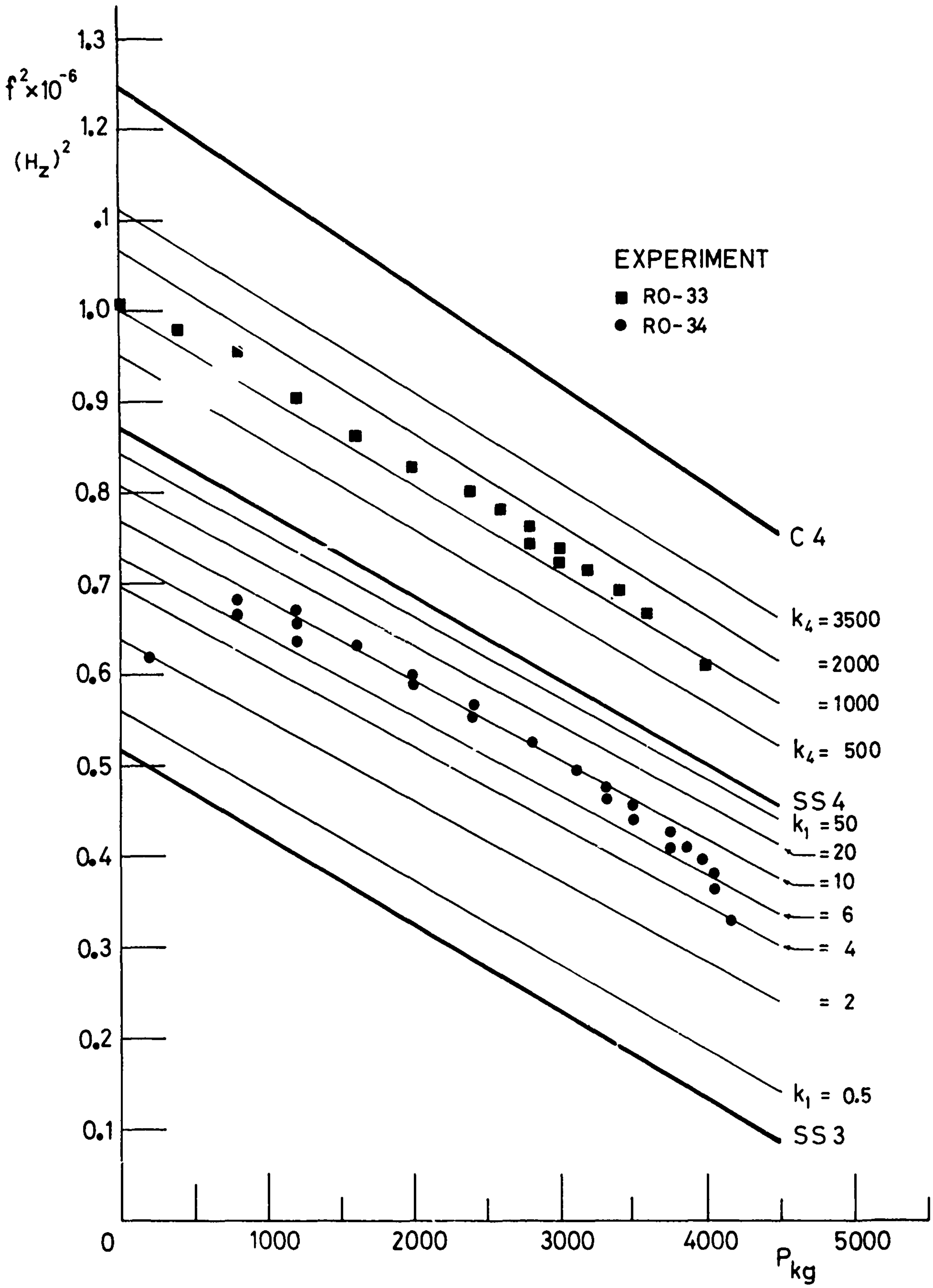

FIG. 8 FREQUENCY SQUARED VERSUS AXIAL LOAD-SHELLS RO-33 AND RO-34 FOR n=7 m=1.

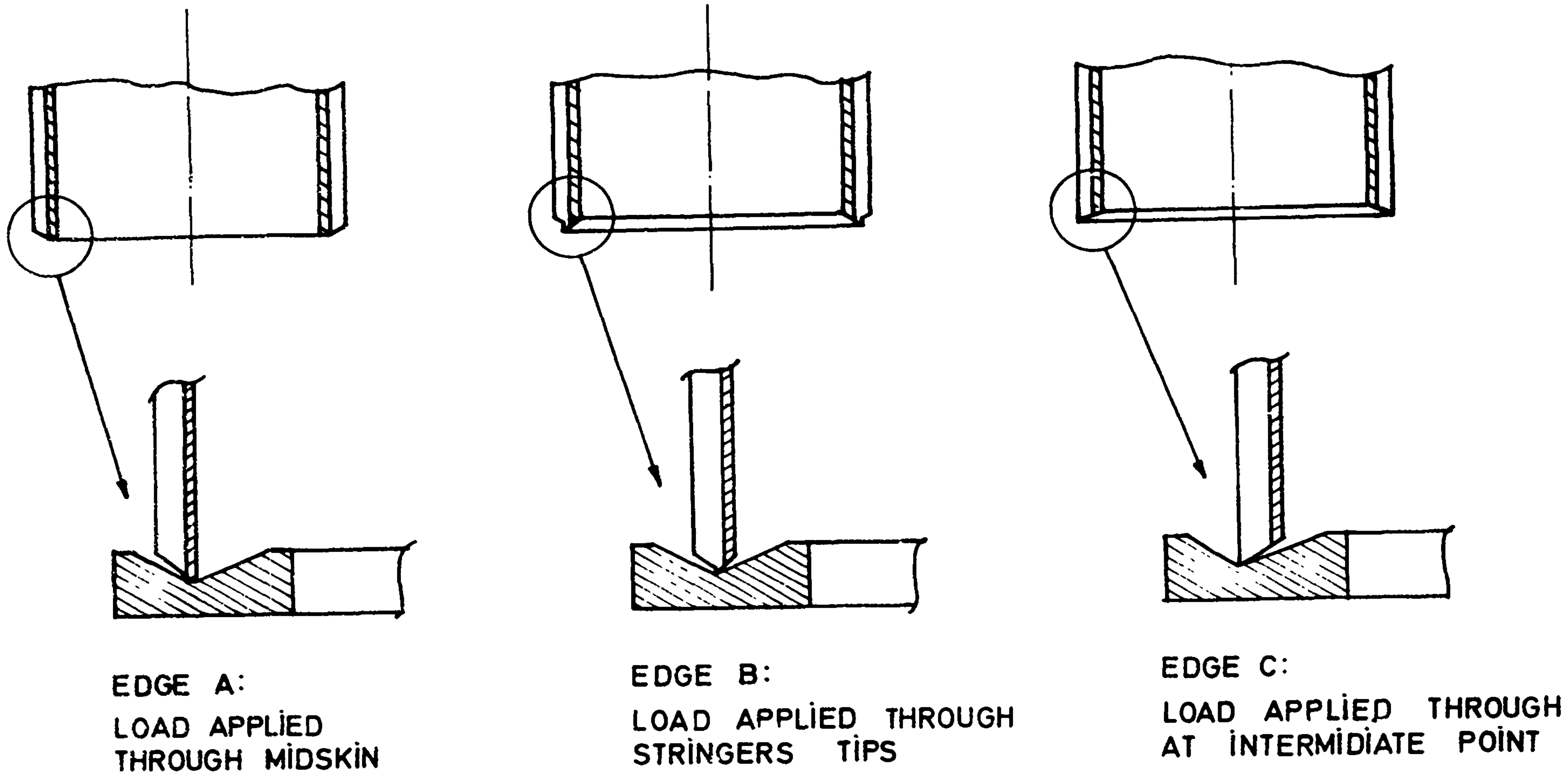

FIG. 9 DETAILS OF LOAD APPLICATION
FOR DIFFERENT TYPES OF EDGES

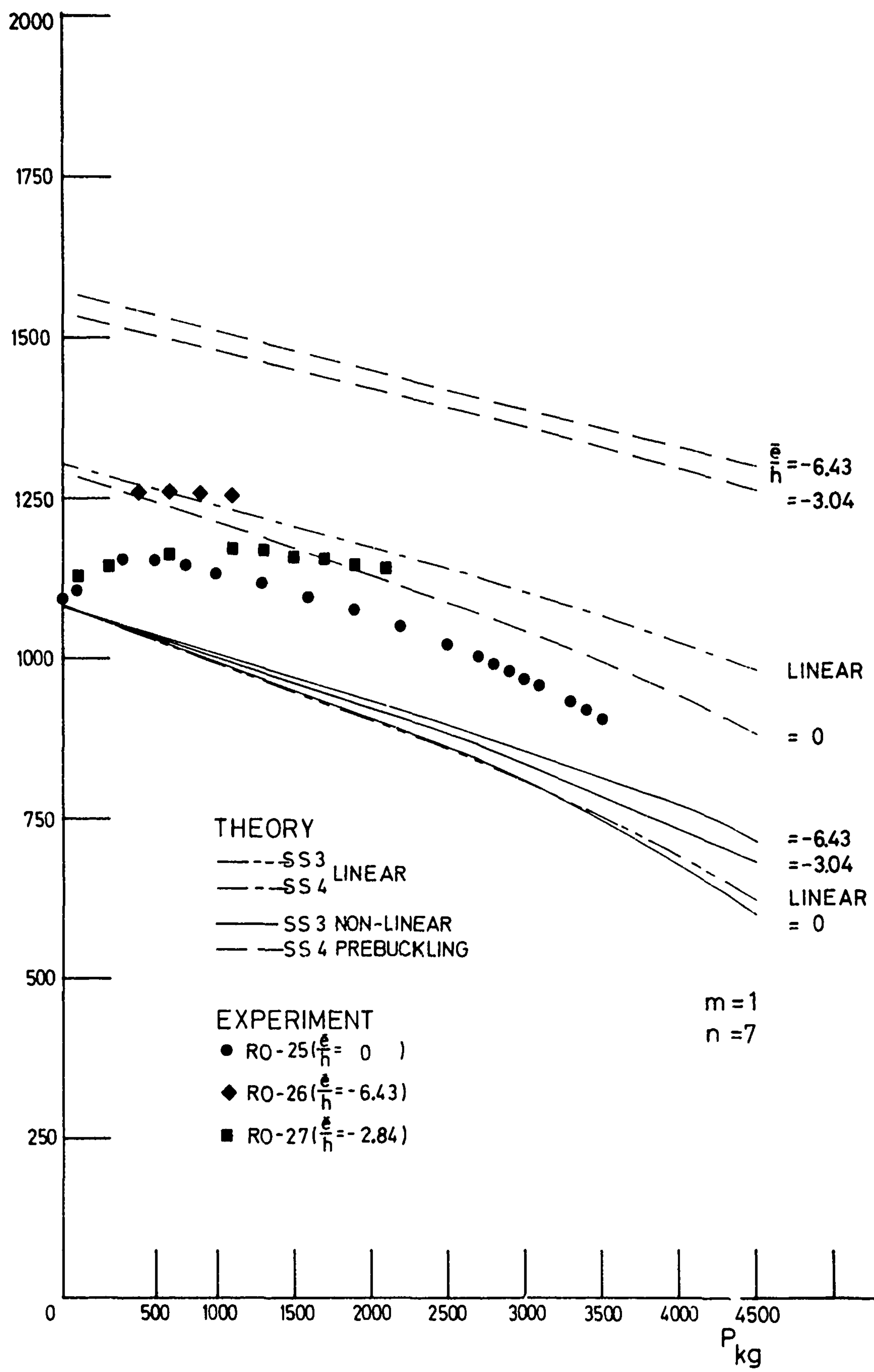

FIG. 10 FREQUENCY VERSUS AXIAL LOAD, "HEAVY" STRINGERS
($A_1/b_1h = 0.59$)

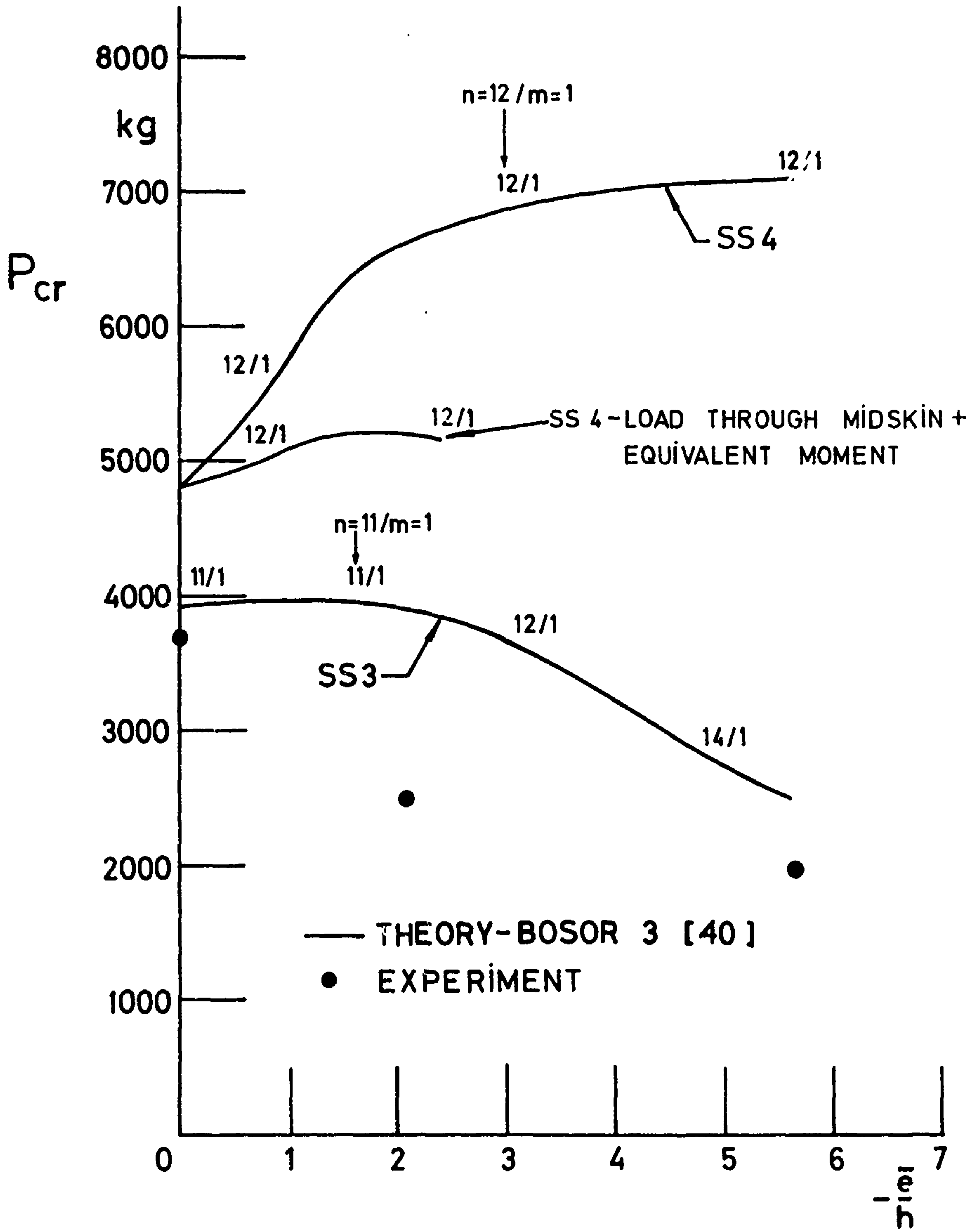

FIG. 11 INFLUENCE OF ECCENTRICITY OF LOADING ON BUCKLING LOADS, "HEAVY" STRINGERS ($A_1/b_1h=0.59$)

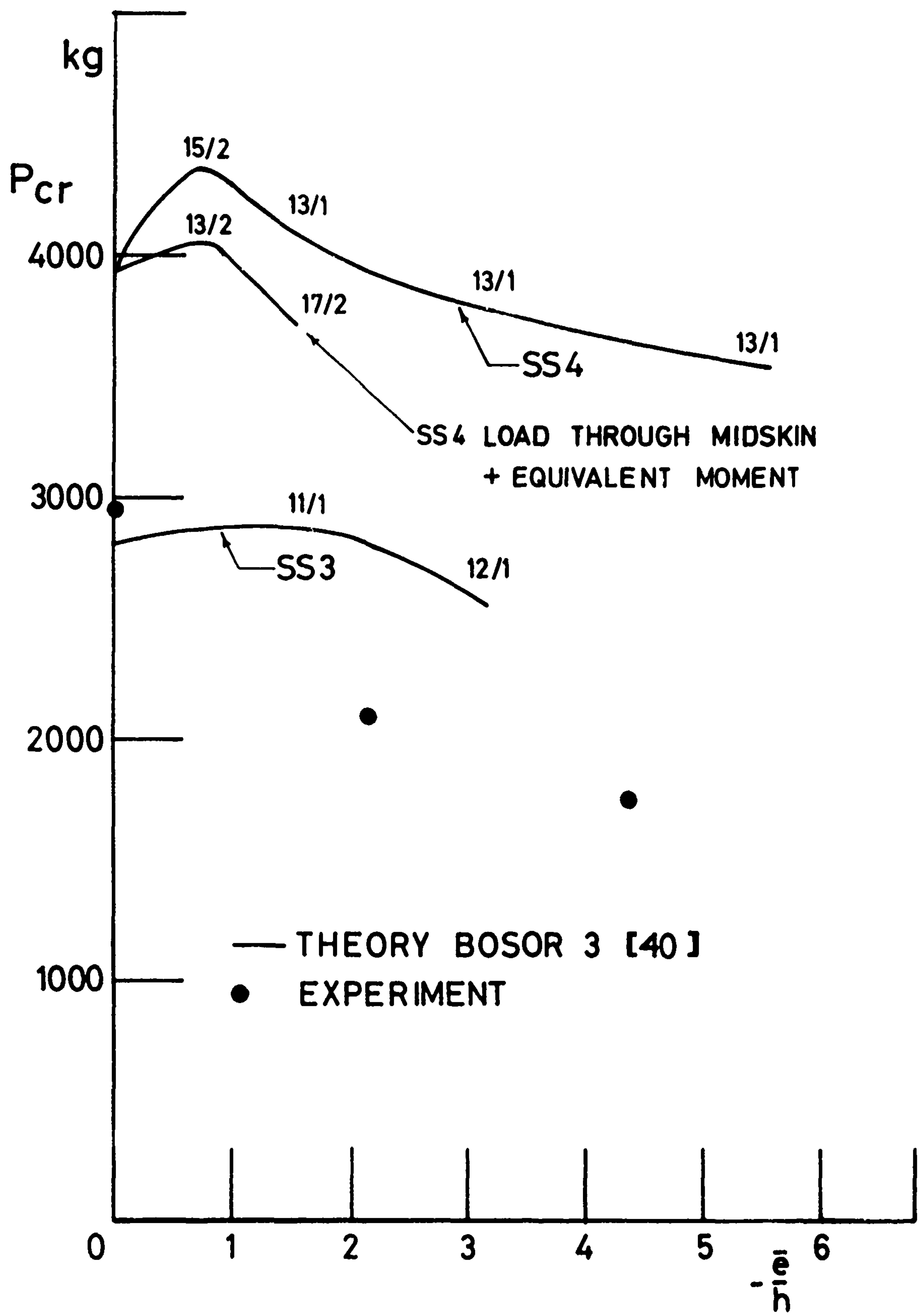

FIG.12 INFLUENCE OF ECCENTRICITY OF LOADING ON BUCKLING LOADS, "MEDIUM" STRINGERS ($A_1 / b_1 h = 0.38$)

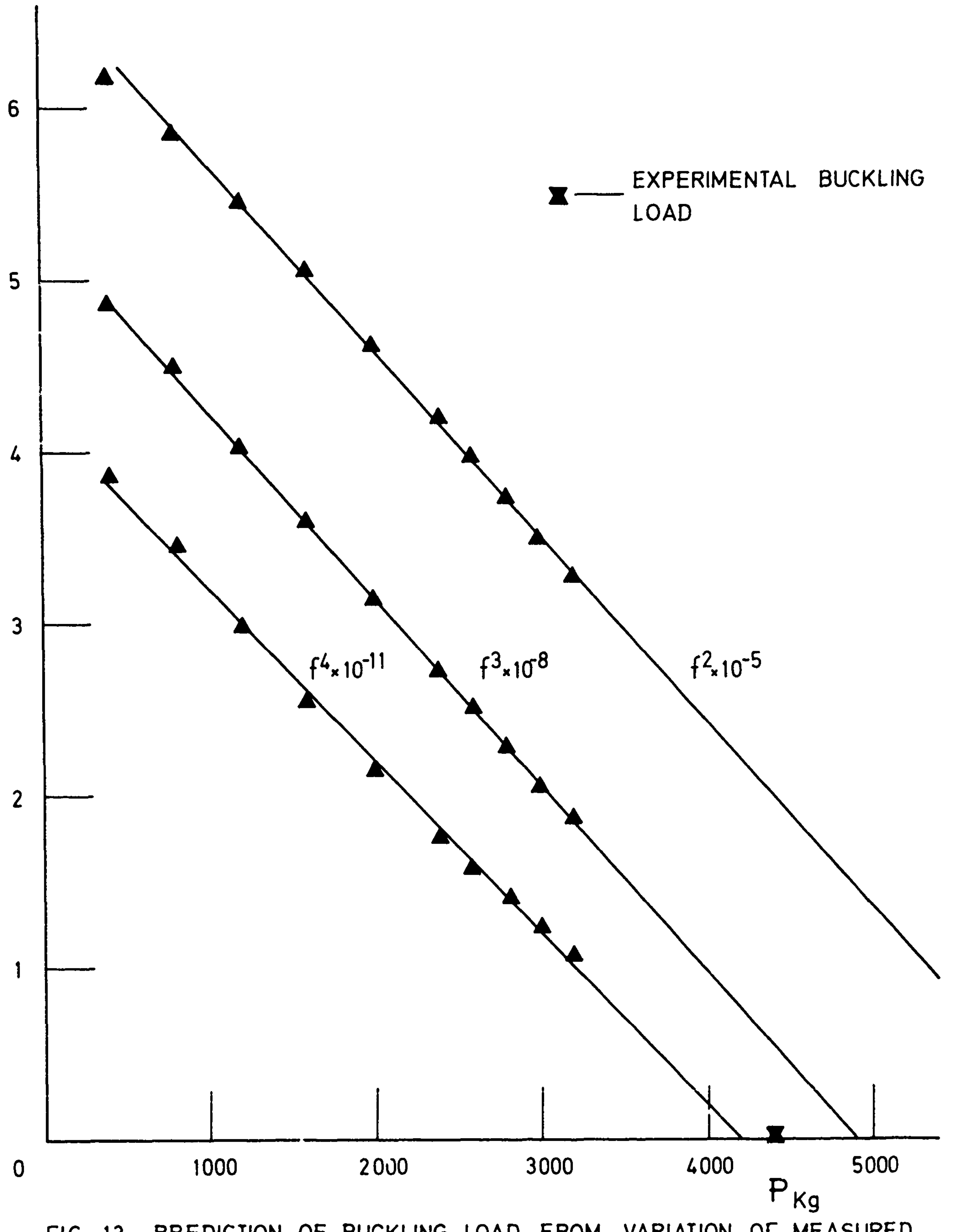

FIG. 13 PREDICTION OF BUCKLING LOAD FROM VARIATION OF MEASURED FREQUENCIES WITH AXIAL LOAD — SHELL RO-32

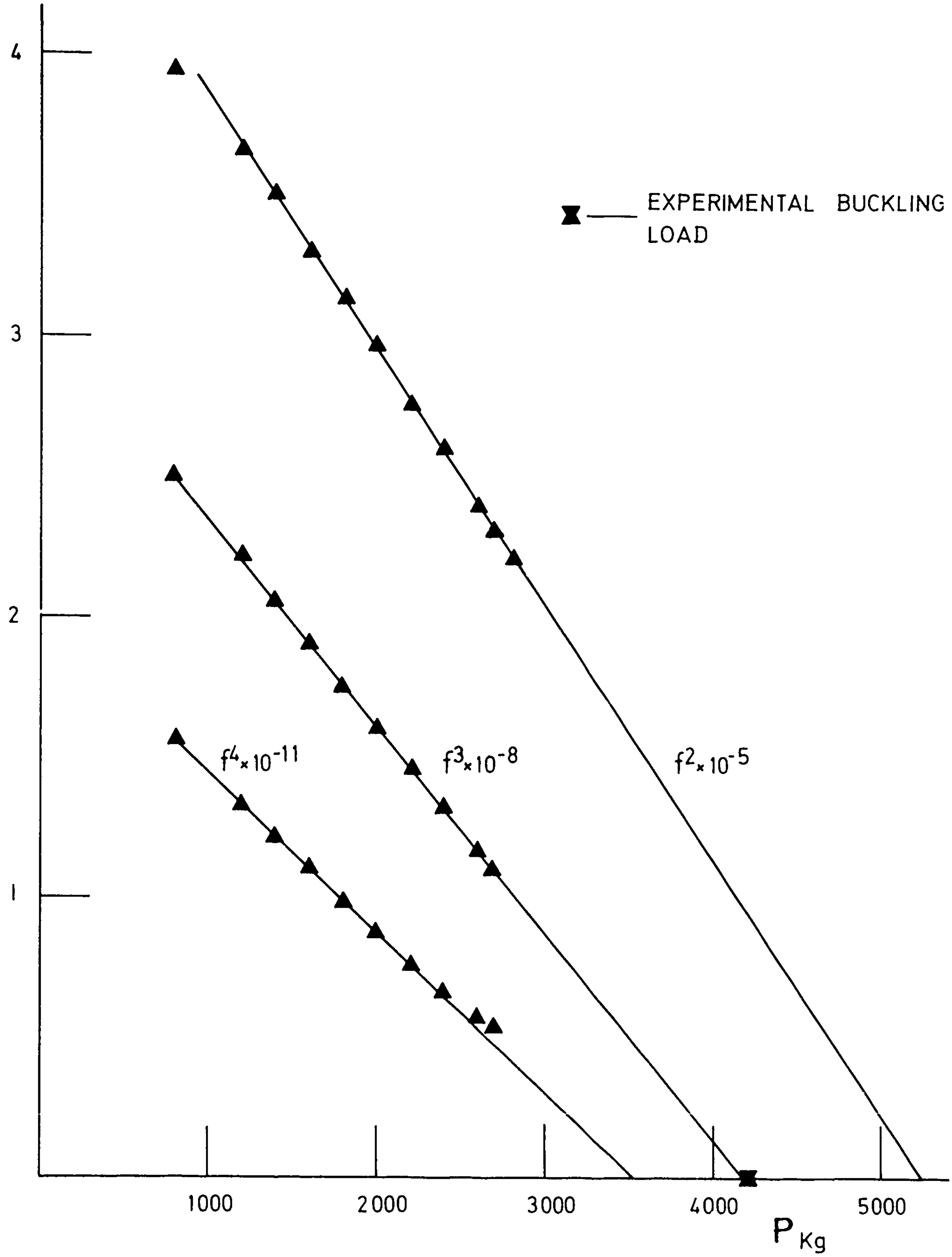

FIG. 14 PREDICTION OF BUCKLING LOAD FROM VARIATION OF MEASURED FREQUENCIES WITH AXIAL LOAD — SHELL RO-34